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## Interaction of Electromagnetic H-wave with the thin Metal Film is Located on the Dielectric Substrate

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Interaction of electromagnetic H-wave with thin metal film is located between two dielectric environments  $\varepsilon_1, \varepsilon_2$  in the case of different incident angles of H-wave  $\theta$  and in the case of different reflection coefficients  $q_1$  и  $q_2$  is calculated in this article. Behavior analysis of reflection coefficient  $R$ , transmission coefficient  $T$  and absorption coefficient  $A$  in the case of its frequency dependence  $\gamma$  and variation dielectric permeability of its environments is done.

**Keywords:** the thin metal film, electromagnetic H-wave, dielectric environments, reflection coefficient, transmission coefficient, absorption coefficient.

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### Introduction

Currently microelectronics, optoelectronics and thin-film technology are actively developing. In particular the greatest interest represents researching of interaction electromagnetic radiation with thin conductive films in the different frequency range [1-6]. This interest is related not only with extensive practical importance of thin conductive films, but with some unresolved theoretical tasks.

In our case thickness of the thin metal film  $a$  is not more, than thickness of skin-layer  $\delta$  and this thickness comparable with the average free path of electrons  $\Lambda$ . For this reason skin-effect is not considered. Skin-effect was researched in [8] in the case of the thin metal cylindrical wire. Quantum effects are not taken into account. This effects were researched in [9] in the case of quantum film in the dielectric environment.

### I. Problem Definition and Methods

Consider the thin metal layer thickness of  $a$  is located between two dielectric (non-magnetic) environments, with dielectric permeability  $\varepsilon_1$  (the first environment) and  $\varepsilon_2$  (the second environment) with reflection coefficients  $q_1$  and  $q_2$  in the case of falling electromagnetic H-wave (from the first environment) at the  $\theta$  angle. Reflection coefficients  $q_1$  and  $q_2$  are associated with reflection of electrons from top and lower surfaces layer. Clarify, if electric field vector is parallel the surface of the thin layer, then this wave is called H-wave. Electric field of electromagnetic wave is parallel the thin metal layer and directed along Y-axis,

while X-axis is directed into the layer.

Then behavior of electromagnetic field inside the thin metal layer is described by the equation system [10]:

$$\begin{cases} \frac{dE_y}{dx} - ikH_z = 0, \\ \frac{dH_z}{dx} + ik(\sin^2 \Theta - 1)E_y = -\frac{4p}{c} j_y. \end{cases} \quad (1)$$

$k = \omega/c$  – wave number,  $c$  – speed of light,  $j$  – electric current density.

We have reflections coefficient  $R$ , transmission coefficient  $T$  and absorption coefficient  $A$  of thin metal film, when H-wave falling on this film [11]:

$$\begin{aligned} T &= \frac{1}{4} \left| P^{(1)} - P^{(2)} \right|^2, \\ R &= \frac{1}{4} \left| P^{(1)} + P^{(2)} \right|^2, \end{aligned} \quad (2)$$

$$A = 1 - T - R.$$

Expression (2) contains  $P^{(1)}$  and  $P^{(2)}$  [12]:

$$\begin{aligned} P^{(1)} &= \frac{\sqrt{e_1} \cdot Z^{(1)} \cos \Theta - 1}{\sqrt{e_1} \cdot Z^{(1)} \cos \Theta + 1}, \\ P^{(2)} &= \frac{\sqrt{e_1} \cdot Z^{(2)} \cos \Theta - 1}{\sqrt{e_1} \cdot Z^{(2)} \cos \Theta + 1}. \end{aligned} \quad (3)$$

$Z^{(1)}$  and  $Z^{(2)}$  correspond to the impedance of lower surface the layer. In particular  $Z^{(1)}$  corresponds antisymmetric, in electric field, configuration of the external field:  $E_y(0) = -E_y(a)$ ,  $H_z(0) = H_z(a)$ , and  $Z^{(2)}$  – corresponds symmetric

configuration:  $E_y(0) = E_y(a)$ ,  $H_z(0) = -H_z(a)$  [11].

Expression for surface impedance in the case of interaction H-wave with the thin metal film, were obtained in [11] in the case when wavelength much more thickness of the thin layer:

$$\begin{aligned} Z^{(1)} &= 0, \\ Z^{(2)} &= \frac{c}{2\mu_0 s_a} \end{aligned} \quad (4)$$

For  $\sigma_a$  expression (this is electrical conductivity of the thin metal layer, with average thickness of this layer) we used results [13]. In this article we compared our results with experiment data [14]. Our  $\sigma_a$  expression have look:

$$\begin{aligned} s_a = s_0 l \int_0^1 (1-t^2) & \left[ 2a + \frac{at}{x-iy} \left[ \frac{q_1 [q_2 \exp(-(x-iy)/t) - \exp(-(x-iy)/t) + 1] - 1}{q_1 q_2 \exp(-2(x-iy)/t) - 1} + \right. \right. \\ & \left. \left. + \frac{q_2 [q_1 \exp(-(x-iy)/t) - \exp(-(x-iy)/t) + 1] - 1}{q_1 q_2 \exp(-2(x-iy)/t) - 1} \right] \left[ \exp(-(x-iy)/t) - 1 \right] dt \right] \end{aligned} \quad (5)$$

$x = a/(v_F \tau)$  – the dimensionless frequency of bulk electron collision,  $y = a\omega/v_F$  – the dimensionless frequency of the electric field,  $\lambda = x/(x-iy)$ ,  $\sigma_0 = \omega_p^2 \tau / 4\pi$  – the static electrical conductivity,  $v_F$  – Fermi speed,  $\tau$  – electron relaxation time,  $\omega_p$  – plasma frequency,  $q_1$  и  $q_2$  –

reflection coefficients.

Finally, reflection coefficient  $R$ , transmission coefficient  $T$  and absorption coefficient  $A$  (expression (2)) will have look [12]:

$$\begin{aligned} R &= \left| \frac{\sqrt{e_{1,2} - \sin^2 \Theta} (\bar{P} + P^{(1)} P^{(2)}) + \cos \Theta (\bar{P} - P^{(1)} P^{(2)})}{\sqrt{e_{1,2} - \sin^2 \Theta} (1 + \bar{P}) + \cos \Theta (1 - \bar{P})} \right|^2 \\ T &= \cos \Theta \operatorname{Re} \sqrt{e_{1,2} - \sin^2 \Theta} \left| \frac{P^{(2)} - P^{(1)}}{\sqrt{e_{1,2} - \sin^2 \Theta} (1 + \bar{P}) + \cos \Theta (1 - \bar{P})} \right|^2 \\ A &= 1 - T - R. \quad \varepsilon_{1,2} = \varepsilon_2 / \varepsilon_1, \quad \bar{P} = \frac{1}{2} (P^{(1)} + P^{(2)}). \end{aligned} \quad (6)$$

Now we will begin to analyze behavior of this coefficients (expression (6)).

## II. Results and Discussion

Let us consider behavior of coefficients  $R$ ,  $T$  and  $A$  in the case of their frequency dependence with variation dielectric permeability value of the second environment  $\varepsilon_2$  and in the case of different reflection coefficients  $q_1$  and  $q_2$ . Clarify some parameters of potassium for further calculations:  $\omega_p = 6.5 \cdot 10^{15}$  1/s,  $v_F = 8.52 \cdot 10^5$  m/s,  $\tau = 1.54 \cdot 10^{-13}$  s,  $a = 10$  nm.

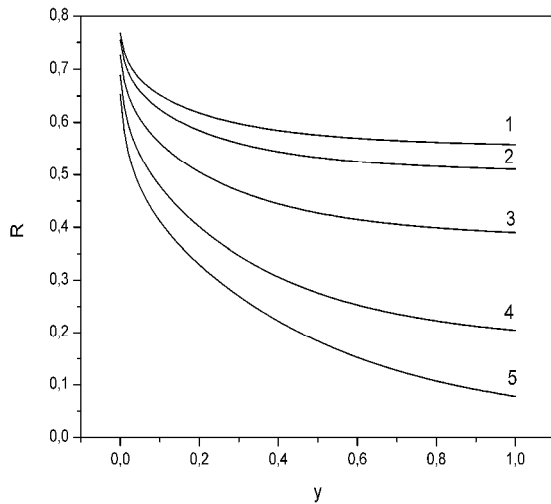
## Conclusions

In figure 1 we can see that the descending velocity of the curve increases with increasing values of the dielectric permittivity of the second environment  $\varepsilon_2$ .

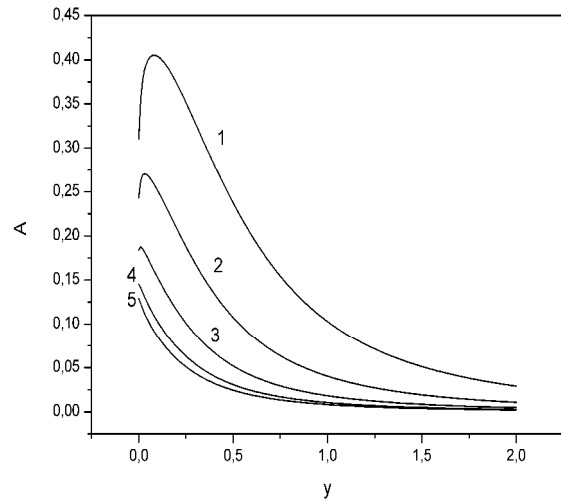
In figure 2 we can see that the increase velocity of the curve increases with increasing values of the dielectric permittivity of the second environment  $\varepsilon_2$ .

In figure 3, in the case of not large value of the dielectric permittivity ( $\varepsilon_2 < 30$ ) we can see, that coefficient  $A$  increases, reaches maximum and descents. Clearly visible absorption maxima. In the case of large value of the dielectric permittivity ( $\varepsilon_2 > 30$ ) coefficient  $A$  begin immediately descents.

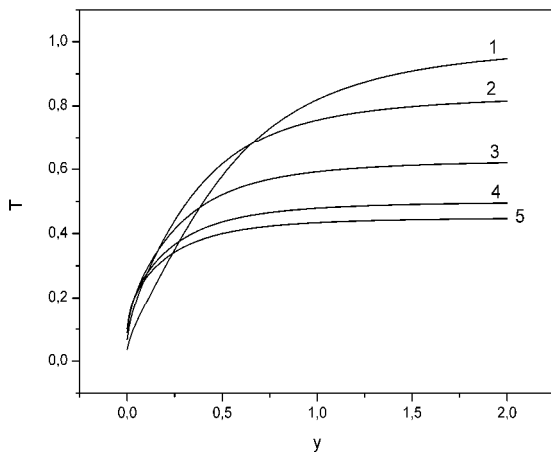
In figure 4, 5, 6 we can see, that variation of the thin



**Fig. 1.** The dependence of reflection coefficients  $R$  on the dimensionless frequency of the electric field  $y$ . Curve 1:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 40$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 2:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 30$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 3:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 15$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 4:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 5$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 5:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 1$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ .



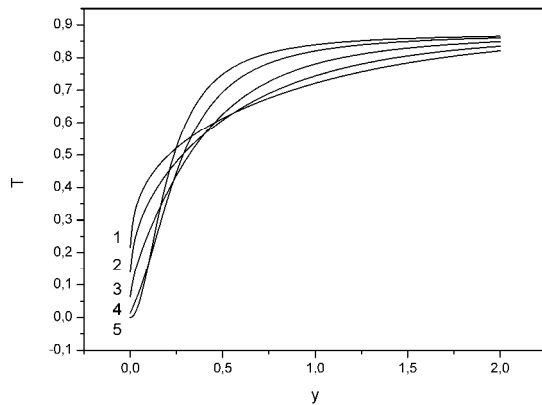
**Fig. 3.** The dependence of absorption coefficients  $A$  on the dimensionless frequency of the electric field  $y$ . Curve 1:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 1$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 2:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 5$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 3:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 15$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 4:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 30$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 5:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 40$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ .



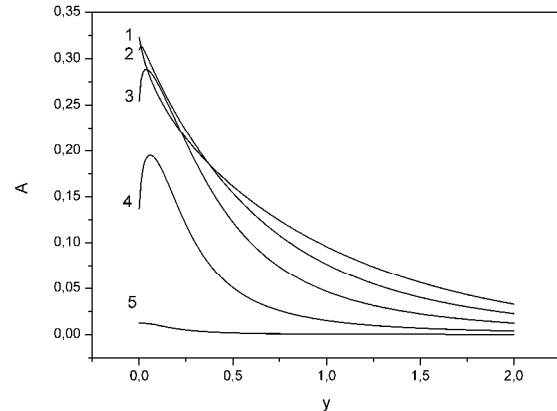
**Fig. 2.** The dependence of transmission coefficients  $T$  on the dimensionless frequency of the electric field  $y$ . Curve 1:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 1$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 2:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 5$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 3:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 15$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 4:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 30$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 5:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 40$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ .



**Fig. 4.** The dependence of reflection coefficients  $R$  on the dimensionless frequency of the electric field  $y$ . Curve 1:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 1$ ,  $q_2 = 1$ ; curve 2:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0.8$ ,  $q_2 = 0.9$ ; curve 3:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 4:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0.2$ ,  $q_2 = 0.3$ ; curve 5:  $x = 0.002$ ,  $\theta = 20^0$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0$ ,  $q_2 = 0$ .



**Fig. 5.** The dependence of transmission coefficients  $T$  on the dimensionless frequency of the electric field  $y$ . Curve 1:  $x = 0.002$ ,  $\theta = 20^\circ$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0$ ,  $q_2 = 0$ ; curve 2:  $x = 0.002$ ,  $\theta = 20^\circ$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0.2$ ,  $q_2 = 0.3$ ; curve 3:  $x = 0.002$ ,  $\theta = 20^\circ$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 4:  $x = 0.002$ ,  $\theta = 20^\circ$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0.8$ ,  $q_2 = 0.9$ ; curve 5:  $x = 0.002$ ,  $\theta = 20^\circ$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 1$ ,  $q_2 = 1$ .



**Fig. 6.** The dependence of absorption coefficients  $A$  on the dimensionless frequency of the electric field  $y$ . Curve 1:  $x = 0.002$ ,  $\theta = 20^\circ$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0$ ,  $q_2 = 0$ ; curve 2:  $x = 0.002$ ,  $\theta = 20^\circ$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0.2$ ,  $q_2 = 0.3$ ; curve 3:  $x = 0.002$ ,  $\theta = 20^\circ$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0.5$ ,  $q_2 = 0.6$ ; curve 4:  $x = 0.002$ ,  $\theta = 20^\circ$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 0.8$ ,  $q_2 = 0.9$ ; curve 5:  $x = 0.002$ ,  $\theta = 20^\circ$ ,  $\varepsilon_1 = 1$ ,  $\varepsilon_2 = 4$ ,  $q_1 = 1$ ,  $q_2 = 1$ .

metal layer reflection coefficients  $q_1$  and  $q_2$  (from diffuse  $q_1 = q_2 = 0$  to reflection  $q_1 = q_2 = 1$  cases) affects to  $R$ ,  $T$ ,  $A$  coefficients. It is obvious that reflection coefficients will change, when the thin metal film borders with different environments. In particular, in figure 6, in the reflection case, coefficient  $A$  immediately descents. In all other cases coefficient  $A$  increases reaches its maximum and descents.

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