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APPLICATION OF DUALITY THEORY TO SOLVE TWO-CRITERIA PROBLEM OF LINEAR PROGRAMMING FOR ECOLOGICAL-ECONOMIC SYSTEM

In the paper, we investigate two-criterion optimization problem: maximization of one target function and minimization of another target function. To solve the offered two-criterion problem, the method of the main criterion is applied. We consider the problem of production activity of the ecological-economic system with the maximization of the value of the final product as the first target function and the minimization of emissions of polluters into the environment as the second target function. We constructed of two production functions (economic and ecological). To construct the economic production function, we select maximal producing of the final products in a costing form as the most essential (main) criterion. Also, there is introduced the appropriate data of the criterion level total volume of emissions of polluters into the environment. After this two-criteria problem is reduced to one - criteria problem. For the construction of ecological production function, the main criterion in the problem of the minimal general volume of emissions of polluters into the environment is defined. We use a parameter of the criterion level of the second criterion and obtained one-criterion problem. Therefore, investigation of the appropriate dual problems explicitly provides economic and ecological production functions to the deduced one-criterion problems. These functions in input two-criterion problem give way to optimal manage of ecological-economic system.

Key words and phrases: Optimal management, two-criterion problem, dual problem, target function, main criterion method, ecological-economic equilibrium, production function.

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INTRODUCTION

Public production envisages not only a creation of material welfares but also activity connected with a decrease of environmental pollution and restoring of natural resources. High level of competitiveness in West European industries as well as a new policy of sustainable development causes a rethinking of management strategy of individual countries.

One of basic tasks, which a modern specialist-economist must be able to solve, is providing of the state, when economic and ecological requirements are balanced. That means providing equilibrium state of ecological economy. The ecological economy is a market economy that studies interaction between ecosystems, social associations and economic systems and also conditions that provide a steady, inexhausting state and progressive development of all three systems. The main task of the ecological economy is forming of fundamentally new directions of transformation of economy based on permanent recreation of innovative processes of reformation of production and consumption of products. The final goal of ecological economy is the sequential passing to ecologically more perfect production technologies, types of

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products, processes of consumption, economic relations, lifestyle [11]. At consideration of ecological economy activity of production can be presented as an ecological-economic production function in which, next to economic values ecological factors also have their market estimation. There is a necessity to build such a production function that has two constituents: economic and ecological.

Analysis of the last researches and publications

A modeling of the ecological-economic systems is the topic of many researches. The ecological-economic models by a structure can be divided into balance, optimization and simulation models. The balance models include the inter-branch Leontief-Ford model of the economy structure impact on the environment [6], and also its generalizations such as the Ayres-Kneese model [1] and the Willen model [23]. At the fact, the Willen model is a matrix balance in the Ayres-Kneese model. Among the simulation models we should mention the Meadows model [10] and the Forrester model [3]. J.W. Forrester attempted to analyze the interactions of demographic, industrial and agrarian systems. In the optimization ecological-economic models, the formation of the optimality criterion is fundamental. Particularly, I. Schimazu [21] proposed optimality criterion, which corresponds to three stages of society development. H. E. Dali [2] and T. H. Tietenberg [22] also considered ecological component in their models. Other scientists [12, 18–20] investigated regulation of the ecological consequences of economic growth and improvements for environmental management.

Among the Ukrainian scientists we have to mark works of I.Lyashenko. Exactly his school works actively on the problems of ecological-economic modeling. In spite of the large volume of publications, plenty of questions and problems for today are not studied enough, and the problem of the balanced economic, social and ecological development remains unsolved.

Now the problems of ecological-economic equilibrium are actively studied.

In [24, p. 446–455] the problem of manufactures ecologization is analyzed and the expansion of the classical model of interbranch balance and its transformation into the optimization model by inclusion of restrictions on emissions is carried out. This model is completed by marginal variables and by corresponding to them coefficients.

In [14, p. 931–938] the research of optimal trajectories of development of the ecological-economic system is carried out in case of equable division of labour resources between branches of material and nature protection productions.

In [15, p. 217–221], [16, p. 31] the technological structure of production in the ecological-economic system with taking into account introduction of technological innovations was researched. In particular, on the basis of distribution of production capacities according to technologies a corresponding equalization of dynamics, based on the set initial conditions and limits on economic and ecological resources, was built. The modelling of ecological-economic interaction in the process of realization of Kyoto protocol decisions which was made is very important [13].

In [17, p. 331–333] the parametrization of mathematical models of the ecological - economic systems is carried out in space of indexes of economic structure of society, prices and environmental pollution.

In [5, p. 170] the conditions for optimal interaction between basic and auxiliary productions in the ecological-economic systems have been investigated. The are the basis for prediction the of the investment level into auxiliary production, which provides its possible growth.

In [4, p. 184–185] the dynamic models of single sectorial economy taking into account the utilization processes of the created pollution and socio-economic structurization were developed.

In [8, p. 149–150] a comparative analysis of the most famous models of ecological-economic systems is presented. The approach to modelling of economic systems considering environmental factors, based on a modified model of Leontief-Ford and the principle of input-output is proposed.

The research of ecological-economic equilibrium is necessary nowadays, when the environmental pollution gained the global character. Using of ecological-economic function is very important in the researches as such function will reflect modelling of ecological-economic system.

The aim of our researches is a construction of optimal functioning model for ecological-economic system to find a solution of suitable two-criterion problem. The problem has solution as two ecological-production functions. We simultaneously consider the maximization of value of the final product and the minimization of emissions of polluters. Since it is difficult to create an effective production activity that takes into account not only economic benefits but also an environmental impact, we want to propose an ecological-economic model combining simultaneously these two factors. It is a necessary way of the problem solving.

THE MAIN RESULTS OF RESEARCHES

Let us consider at the ecological-economic system that includes a basic (material) production and auxiliary production (sewage treatment plants). The volumes of productions are limited to the present resources. The central core of the ecological-economic system is an inter-branch model of V. Leontief and D. Ford [7, p. 21]. It represents cooperation of industries of producing of products and industries that destroy harmful wastes. The efficiency of production activity is described by two criteria (economic and ecological). The first criterion is maximal producing of the final products in a cost form. The second criterion is minimal general volume of emissions of polluters into the environment.

We suggest to describe production activity of the ecological-economic system as following two-criteria problem of linear programming:

$$\begin{aligned} &f_1(x) = c^1 y^1 \to \max,\ f_2(x) = c^2 y^2 \to \min, \\ &x^1 = A_{11}x^1 + A_{12}x^2 + y^1,\ x^2 = A_{21}x^1 + A_{22}x^2 - y^2, \\ &B_1 x^1 + B_2 x^2 \le R,\ x^1 \ge 0,\ x^2 \ge 0,\ y^1 \ge 0,\ y^2 \ge 0, \end{aligned} \tag{1}$$

where c^1 is a vector of products' prices, c^2 is a vector of equivalence's coefficients of polluters (in relation to harm or in relation to the cost of destruction), x^1 is a vector of the gross producing of products, y^1 is a vector of producing of final products, x^2 is a vector of volumes of the destroyed polluters, y^2 is a vector of volumes of emissions of polluters in the environment, A_{11} and A_{12} are technological matrices of direct issue of products, A_{21} and A_{22} are technological matrices of the direct producing of pollutants (in a basic production and sewage treatment plants accordingly), B_1 and B_2 are matrices of costs of economic resources for the basic and the auxiliary productions, R is a vector of present economic resources.

The necessary and sufficient condition of non-negativity of solutions of the Leontief-Ford model at the productivity of block matrix $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \geq 0$ and at the $y^1 > 0$, $y^2 \geq 0$ is the condition $x^2 = (E_2 - A_2)^{-1}\left(A_{21}(E_2 - A_2)^{-1} y^1 - y^2\right) \geq 0$, where $A_2 = A_{22} + A_{21}(E_1 - A_{11})^{-1} A_{12}$, and E_1,E_2 are diagonal identity matrices [7, p. 27]. Besides from Leontief-Ford inter-branch balance we can get the next $x^1 = (I_1 - A_{11})^{-1} \cdot (A_{12}x^2 + y^1)$. It implies that if $x^2 \geq 0$ and $y^1 \geq 0$ then $x^1 \geq 0$. So, the productivity of ecological-economic system $\left(x^1 \geq 0, x^2 \geq 0\right)$ will be provided when $\left(y^1 \geq 0, y^2 \geq 0\right)$. Then the two-criterion problem (1) after the exclusion of variables x^1 and x^2 can be the following form:

$$
\begin{aligned}
& f_1(x) = c^1 y^1 \to \max,\ f_2(x) = c^2 y^2 \to \min, \\
& x^1 = A_{11}x^1 + A_{12}x^2 + y^1, \\
& x^2 = (E_2 - A_2)^{-1}\left(A_{21}(E_2 - A_2)^{-1} y^1 - y^2\right) \geq 0, \\
& D_1 y^1 - D_2 y^2 \leq R,\ y^1 \geq 0,\ y^2 \geq 0,
\end{aligned}
\tag{2}
$$

where

$$
\begin{aligned}
D_1 &= B_1(E_1 - A_1)^{-1} + B_2(E_2 - A_2)^{-1} A_{21}(E_1 - A_{11})^{-1} \geq 0, \\
D_2 &= B_1(E_1 - A_1)^{-1} A_{12}(E_2 - A_{22})^{-1} + B_2(E_2 - A_2)^{-1} \geq 0.
\end{aligned}
$$

For the investigation of two-criterion problem (2) we apply the method of main criterion [9, p. 47]. Thus, we build two production functions (economic and ecological).

For the construction of economic production function we will distinguish $f_1(x)$ as the main criterion and we will set admissible value $Z \geq 0$ for the criterion level $f_2(x)$, that is total volume of emissions of polluters into the environment.

We obtain the following one-criterion problem:

$$
\begin{aligned}
& c^1 y^1 \to \max, \\
& c^2 y^2 \leq Z, \\
& (E_2 - A_2)^{-1} A_{21}(E_2 - A_2)^{-1} y^1 - (E_2 - A_2)^{-1} y^2 \geq 0, \\
& D_1 y^1 - D_2 y^2 \leq R,\ y^1 \geq 0,\ y^2 \geq 0.
\end{aligned}
\tag{3}
$$

Let us write the problem (3) in the next way:

$$
\begin{aligned}
& c^1 y^1 \to \max, \\
& c^2 y^2 \leq Z, \\
& -(E_2 - A_2)^{-1} A_{21}(E_2 - A_2)^{-1} y^1 + (E_2 - A_2)^{-1} y^2 \leq 0, \\
& D_1 y^1 - D_2 y^2 \leq R,\ y^1 \geq 0,\ y^2 \geq 0.
\end{aligned}
\tag{4}
$$

The dual problem to problem (3) has the following form:

$$
\begin{aligned}
& pZ + rR \to \min, \\
& -q(E_2 - A_{22})^{-1} A_{21}(E_1 - A_{11})^{-1} y^1 + rD_1 \geq c^1, \\
& pc^2 + q(E_2 - A_{22})^{-1} - rD_2 \geq 0, \\
& p \geq 0,\ q \geq 0,\ r \geq 0,
\end{aligned}
\tag{5}
$$

where $p \geq 0, q \geq 0, r \geq 0$ are dual variables (the cost of emissions of polluters into the environment, the cost of the destroyed polluters, the cost of economic resources).

For problem (5) we will find the basic feasible solution from the auxiliary system of linear equations:

$$\begin{aligned} & -q\left(E_2 - A_{22}\right)^{-1} A_{21}\left(E_1 - A_{11}\right)^{-1} y^1 + rD_1 - \lambda = c^1, \\ & pc^2 + q\left(E_2 - A_{22}\right)^{-1} - rD_2 - \omega = 0, \end{aligned} \tag{6}$$

where $\lambda \geq 0, \omega \geq 0$ are slack variables.

We find all basic feasible solutions of system (6), namely $\left(p_1^*, q_1^*, r_1^*\right), \left(p_2^*, q_2^*, r_2^*\right), \ldots,$ $\left(p_s^*, q_s^*, r_s^*\right)$. Then we explicitly write down an economic production function

$$F(Z,R) = c^1 y^{1*} = \min(p^* Z + r^* R) = \begin{cases} p_1^* Z + r_1^* R = f_1^1\left(x(Z,R)\right), x \in M_1, \\ \ldots \quad \ldots \quad \ldots \\ p_s^* Z + r_s^* R = f_1^s\left(x(Z,R)\right), x \in M_s, \end{cases}$$

where $\bigcup_{i=1}^{s} M_i = M$ is the domain of definition of the problem (3).

A production function depends on the parameter Z and is presented in dependence of the chosen set. At the different values of parameter Z we will obtain the different optimal solutions for the two-criterion problem (1).

For the construction of ecological production function we will define $f_2(x)$ as the main criterion in problem (2) and we will set admissible value $Q \geq 0$ of final producing products in a cost form (the first criterion).

We will get the following one-criterion problem

$$\begin{aligned} & c^2 y^2 \to \min, \\ & c^1 y^1 \geq Q, \\ & \left(E_2 - A_2\right)^{-1} A_{21}\left(E_2 - A_2\right)^{-1} y^1 - \left(E_2 - A_2\right)^{-1} y^2 \geq 0, \\ & D_1 y^1 - D_2 y^2 \leq R,\ y^1 \geq 0,\ y^2 \geq 0. \end{aligned} \tag{7}$$

Let us write problem (7) in the next way:

$$\begin{aligned} & -c^2 y^2 \to \max, \\ & -c^1 y^1 \leq -Q, \\ & -\left(E_2 - A_{22}\right)^{-1} A_{21}\left(E_1 - A_{11}\right)^{-1} y^1 + \left(E_2 - A_{22}\right)^{-1} y^2 \leq 0, \\ & D_1 y^1 - D_2 y^2 \leq R,\ y^1 \geq 0,\ y^2 \geq 0. \end{aligned} \tag{8}$$

The dual problem to problem (8) looks as following:

$$\begin{aligned} & -uQ + wR \to \min, \\ & -uc^1 - v(E_2 - A_{22})^{-1} A_{21}(E_1 - A_{11})^{-1} + wD_1 \geq 0, \\ & v(E_2 - A_{22})^{-1} - wD_2 \geq -c^2, \\ & u \geq 0, v \geq 0, w \geq 0, \end{aligned} \tag{9}$$

where $u \geq 0, v \geq 0, w \geq 0$ are dual variables (the cost of producing of final products, the cost of the destroyed polluters, the cost of economic resources).

For problem (9) we will find the basic feasible solutions from the auxiliary system of linear equations:

$$\begin{aligned} &-uc^1 - v(E_2 - A_{22})^{-1}A_{21}(E_1 - A_{11})^{-1} + wD_1 - \mu = 0, \\ &v(E_2 - A_{22})^{-1} - wD_2 - \nu = -c^2, \end{aligned} \tag{10}$$

where $\mu \geq 0, \nu \geq 0$ are slack variables.

We find all basic feasible solutions of system (10), namely $\left(u_1^*, v_1^*, w_1^*\right), \left(u_2^*, v_2^*,, w_2^*\right), \ldots,$ $\left(u_k^*, v_k^*, w_k^*\right)$. Then we explicitly write down an economic production function

$$F(Q,R) = c^2y^{2*} = \min(-u^*Q + w^*R) = \max(u^*Q - w^*R) =$$
$$= \begin{cases} u_1^*Q - w_1^*R = f_2^1\left(x(Q,R)\right), x \in \tilde{M}_1, \\ \ldots \quad \ldots \quad \ldots \\ u_k^*Q - w_k^*R = f_2^k\left(x(Q,R)\right), x \in \tilde{M}_k, \end{cases}$$

where $\bigcup_{i=1}^k \tilde{M}_i = \tilde{M}$ is the domain of definition of the problem (7).

A production function depends on the parameter Q and in dependence of the chosen set. At the different values of the parameter Q we will get the different optimal solutions for the two-criterion problem (1).

Thus, investigation of the proposed two-criterion production activity model of ecological-economic system (1) is reduced to investigation of a pair of one-criterion problems (3) and (7). Applying theory of duality we obtained a pair of production functions for the ecological-economic system in an explicit form:

$$F(Z,R) = c^1y^{1*} = \min(p^*(Z,R) \cdot Z + r^*(Z,R) \cdot R),$$
$$F(Q,R) = c^2y^{2*} = \max(u^*(Q,R) \cdot Q - w^*(Q,R) \cdot R).$$

Example 1. *To demonstrate the construction of a pair of production functions for the ecological-economic system, we consider the following example. Let* (1) *be a given problem, where* $A_{11} = (0,4)$, $A_{12} = (0,2)$, $A_{21} = (0,2)$, $A_{22} = (0,1)$, $B_1 = (2)$, $B_2 = (1)$, $c^1 = (1)$, $c^2 = (2)$, $R = (R)$. *Then problem* (2) *looks as following*

$$\begin{aligned} &y^1 \to \max,\ y^2 \to \min, \\ &x^1 = 0,4x^1 + 0,2x^2 + y^1, \\ &x^2 = \frac{36}{125}y^1 - \frac{6}{5}y^2 \geq 0, \\ &4y^1 - 2y^2 \leq R,\ y^1 \geq 0,\ y^2 \geq 0. \end{aligned}$$

Setting admissible value $Z = (Z) \geq 0$ *(that is a total volume of emissions of polluters into the environment) we obtain one-criterion problem* (4) *in the following form:*

$$\begin{aligned} &y^1 \to max,\ y^2 \leq Z, \\ &-\frac{36}{125}y^1 + \frac{6}{5}y^2 \leq 0, \\ &4y^1 - 2y^2 \leq R,\ y^1 \geq 0,\ y^2 \geq 0. \end{aligned}$$

Let us write a dual problem to the problem:

$$\begin{aligned} &pZ + rR \to \min, \\ &-\frac{36}{125}q + 4r \geq 1, \\ &p + \frac{6}{5}q - 2r \geq 0, \\ &p \geq 0,\ q \geq 0,\ r \geq 0. \end{aligned}$$

Next, we construct an auxiliary system of linear equations and find the basic feasible solutions of the problem

$$-\frac{36}{125}q+4r-\lambda=1,$$
$$p+\frac{6}{5}q-2t-\omega=0.$$

To solve the system we examine it in these cases 1) $p\geq 0, q\geq 0$*; 2)* $p\geq 0, r\geq 0$*; 3)* $p\geq 0, \lambda\geq 0$*; 4)* $p\geq 0, \omega\geq 0$*; 5)* $q\geq 0, r\geq 0$*; 6)* $q\geq 0, \lambda\geq 0$*; 7)* $q\geq 0, \omega\geq 0$*; 8)* $r\geq 0, \lambda\geq 0$*; 9)* $r\geq 0, \omega\geq 0$*; 10)* $\lambda\geq 0, \omega\geq 0$*. A feasible solution can be obtained in the cases 2 and 5. As consequence, there is the following economic production function* $F(Z,R)=\min\{\frac{1}{2}Z+\frac{1}{4}R;\frac{375}{164}R\}$*.*

We set $Q=Q(R)\geq 0$ *as an admissible value of final producing products in a cost form. Then problem* (7) *has form*

$$y^2\to\min,\ y^1\geq Q,$$
$$\frac{36}{125}y^1-\frac{6}{5}y^2\geq 0,$$
$$4y^1-2y^2\leq R,\ y^1\geq 0,\ y^2\geq 0.$$

It can be rewritten as following

$$-y^2\to\max,\ -y^1\leq -Q,$$
$$-\frac{36}{125}y^1+\frac{6}{5}y^2\leq 0,$$
$$4y^1-2y^2\leq R,\ y^1\geq 0,\ y^2\geq 0.$$

Find a dual problem to it

$$-uQ+wR\to\min,$$
$$-u-\frac{36}{125}v+4w\geq 0,$$
$$\frac{6}{5}v-2w\geq -1,$$
$$u\geq 0,\ v\geq 0,\ w\geq 0.$$

To find non-trivial feasible solutions of the system

$$-u-\frac{36}{125}v+4w-\mu=0,$$
$$\frac{6}{5}v-2w-\nu=-1$$

we investigate the cases 1) $u\geq 0, v\geq 0$*; 2)* $u\geq 0, w\geq 0$*; 3)* $u\geq 0, \mu\geq 0$*; 4)* $u\geq 0, \nu\geq 0$*; 5)* $v\geq 0, w>0$*; 6)* $v\geq 0$*;* $\mu\geq 0$*; 7)* $v\geq 0, \nu\geq 0$*; 8)* $w\geq 0, \mu\geq 0$*; 9)* $w\geq 0, \nu\geq 0$*; 10)* $\mu\geq 0, \nu\geq 0$*. It is possible to solve the problem only in cases 2 та 8. As a result, we obtain an ecological production function* $F=\min\{-2Q+\frac{1}{2}R;\frac{1}{2}R\}$*. Thus, there is a constructed pair of productions functions for the ecological-economic system* $F(Z,R)=\min\{\frac{1}{2}Z+\frac{1}{4}R;\frac{375}{164}R\}$*,* $F(Q,R)=\min\{-2Q+\frac{1}{2}R;\frac{1}{2}R\}$*.*

Conclusions. The proposed model of the ecological-economic system gives an opportunity to treat the production activity as the problem of maximization of the final product value and as the problem of minimization of polluters emissions into the environment. In both cases, the ecological-economic production functions are constructed. The production functions describe the optimal performance of the ecological-economic system. Our optimization model allows to realize the largest volume of final product output, the best distribution of economic resources and the least amount of polluters emissions into the environment. The proposed approach to construct a pair of production functions best suits needs and is convenient for usage. Analysis of this pair of production functions provides acceptance of better and more effective decisions at the production management.

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У статті досліджено оптимізаційну задачу з двома критеріями: максимізація однієї цільової функції та мінімізація іншої цільової функції. Для розв'язання запропонованої двокритерійної задачі застосовано метод головного критерію, причому розглянуто задачу виробничої діяльності еколого-економічної системи, в якій реалізується максимізація вартості кінцевого продукту, як перша цільова функція, і мінімізація викидів забруднювачів у навколишнє середовище, як друга цільова функція. Внаслідок цього здійснюється побудова двох виробничих функцій (економічної та екологічної). Для побудови економічної виробничої функції за найбільш суттєвий (головний) критерій виділено максимум випуску кінцевої продукції у вартісній формі й введено параметр із значенням експертно встановленого ″порогового″ (задовільного) рівня іншого критерію. Після цього двокритеріальну задачу приведено до однокритеріальної задачі. Аналогічно, для побудови екологічної виробничої функції за найбільш суттєвий (головний) критерій виділено мінімум загального обсягу викидів забруднювачів у навколишнє середовище, введено параметр ″порогового″ рівня другого критерію і отримано однокритеріальну задачу. Шляхом дослідження відповідних двоїстих задач до отриманих однокритерійних задач у явному вигляді записано економічну та екологічну виробничі функції. Такі функції відносно первинної двокритерійної задачі дають змогу здійснювати оптимальне управління еколого-економічною системою.

Ключові слова і фрази: Оптимальне управління, двокритерійна задача, двоїста задача, цільова функція, метод головного критерію, еколого-економічна рівновага, виробнича функція.