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## Use of Multiple Linear Regression for the Prediction of Thermal Regime of Wells

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Questions of prediction of the thermal regime of a well for the calculation of the drilling parameters have been discussed. Researches and publications have been analyzed, and it has been noted, that despite the fact that the need for such forecast is indicated, there are no recommendations for the techniques in literature sources. It has been proposed to perform this prediction using multiple regression analysis, using the results of the temperature measurements, which are performed during geophysical research of a well immediately after drilling. The example of the use of the proposed methodology has been shown. Based on mathematical statistical analysis of the obtained results, it has been confirmed that the temperature regime depends not only on the depth of the horizontal position, but also on the time of operation of a field.

**Keywords:** field, well, geothermal step, multiple linear regression, prediction.

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### Introduction

During drilling a well due to heat and mass transfer by drilling fluid, temperature of the walls of a well changes stepwise (decreases) leading to thermal stresses. These stresses should be considered during drilling, because they facilitate rock breaking by a chisel, especially where there is a high geothermal gradient.

At the same time temperature determines the choice of the type of solution or binder, the method of chemical treatment, because the physical and mechanical, and especially the rheological properties of drilling and plugging cement mortars more dependent on temperature than even on pressure [6]. Temperature changes hydraulic losses, time of pumping plugging solutions, cement stability of circulating solution etc.

To determine the temperature of drilling fluid after quite a long period of circulation –  $t_{i,well}$ , it is proposed [5] to use the formula:

$$t_{i,well} = \frac{t_{i,static}}{[\sqrt[6]{Q(0.58+10^{-4}Di)}]} \quad (1)$$

$t_{i,static}$  is static temperature in a mine 6-8 hours after circulation stopped.  $Q$  is consumption of drilling fluid, l/s.  $D$  is the depth of a hollow.

First studies of the thermal regime of wells in different regions were carried out by D.V. Holubiatnikov, M.V. Abramovich, V.N. Dahnov and others. Based on

these studies it was recommended for most regions to accept geothermal step (GTS) equal to 33 m/°C. Based on further research [1] it was recommended for determining the change of temperature with the change of depth to use the formula:

$$t_D = 23.5 + 0.0179 D. \quad (2)$$

Note, that  $t_D$  is temperature of a pool at depth  $D$  in a well, drilling of which was completed a few months ago. There are other hypotheses about the distribution of  $t_D$ , which are also based on experimental data [7]. In [2], where the thermal processes in the annulus of a well, that is being drilled, are discussed in details, based on in-depth analysis of the issue it was concluded that for each specific field the values of geothermal gradient (GTG) should be clarified. Especially it refers to determining  $t_{i,static}$ . However, there are no recommendations to such specification (prediction) in normative or technical literature sources.

If we consider the spatial position of a well as the integrating factor, which reflects the impact of all exogenic and anthropogenic factors on temperature field in and around a well, then for this prediction there is a problem of representation of thermal field of a field in the form of analytical dependence

$$t_{i,static} = f(x, y, H, T), \quad (3)$$

where  $T$  is time of operation of a field from time of collaring of the first well;  $x, y$  are horizontal coordinates

of a well;  $H$  is point absolute height (above sea level).

In the dependence (3), using absolute height parameter instead of depth parameter is explained by the physical nature of GTG. Consideration of time in the mathematical model reflects the fact of the temperature increase of oil field during operation.

### I. Formulation of problem of the research

The purpose of work is to develop a method of obtaining and evaluating the accuracy of function parameters (3) according to temperature measurements in wells immediately after the drilling process.

### II. Presentation of main material

Studies have been carried out using materials of measurements of the temperature field in five wells of a field, horizontal and vertical coordinates of which are given in Table 1.

Graphs of temperature changes with absolute height (above sea level) for all seven of wells are shown in Figure 1. Shown graphs of temperature changes in the wells 506 and 337 are presented to illustrate the fact of raising the temperature of the field during operation and obtained based on the results of measurements in the wells, temperature regime of which is stabilized. The same figure shows the graph of the temperature changes with the change of depth calculated using the formula (1).

Analysis of the graphs has shown that, for example, temperature values in the well No. 337 differ significantly from the results of temperature measurements in closely located (distance is less than 100 m) wells No. 512 and No. 352. For illustrative purposes, temperature graphs in these three closely located wells are presented in Fig. 2.

The wells 337 and 352 are the closest in spatial location. Their temperatures in the mouth and the face almost coincide, however, at the same heights their temperatures vary considerably (for example, at an altitude of -1500 m, their temperatures differ by almost 20°C). This is a good illustration of differences between  $t_D$  and  $t_{i,static}$ , total raising the temperature of a field during operation and the need for specification of the

The geological conditions of the part of the field, where investigated wells are located, are such that lithologic and stratigraphic differentiation of rocks and associated with this natural factors (magma chamber, radioactive decay, oxidation of sulphide ores, salt dissolution, the allocation of gas in the well), which are possible in this area, obviously could not cause such a difference of temperatures in adjacent wells at the same altitude. Only the influence of artificial thermal fields (drilling fluid, solidified cement stone, water injection, and special well heaters, etc.) can be the cause. In our opinion, for this case the only factor, the influence of which can explain a significant temperature difference (reducing temperatures in the wells 512, 501, 546 and 352), is the influence of drilling fluid. The wells 546 and 501 have the closest temperatures, at the same time they are the most distant from each other, which also in addition indicates the homogeneity of the geological conditions of the investigated area of the field.

As we have noted, regression equation must be obtained separately for the group of wells, which temperature regime is stabilized, and for the group of wells, data on temperature regime of which are obtained during carottage, which was conducted immediately after drilling process. In the first case, we get the expression that allows predicting the temperature of the annulus, caused by the influence of exogenic factors. In the second case – the temperature of the well annulus during drilling, which is characterized by  $t_{i,static}$ . For this case, the first group includes only two wells 337 and 506. The second group includes wells 352, 512, 501, 546 and 344.

For the prediction, it is proposed to consider the temperature at the height  $H$  relative to sea level  $t_{i,static}$  as a value that depends linearly on spatial coordinates  $x, y, H$  and time  $T$ :

$$t_{i,static} = \alpha_1 \cdot x + \alpha_2 \cdot y + \alpha_3 \cdot H + \alpha_4 \cdot T + b, \quad H = H_{mouth}, \dots, H_{face}, \quad (3)$$

where  $b, \alpha_1, \alpha_2, \alpha_3, \alpha_4$  are parameters that should be defined.

Matrix recording of equation (3)

$$Y = X\alpha, \quad (4)$$

where  $Y$  is the vector of dependent variable, dimensionality  $k \times 1$ ,  $k$  is the number of points where the temperature was measured in a well;  $X$  is the matrix of independent variables ( $x, y, H, T$ ), dimensionality  $k \times 4$ ;  $\alpha$  is the vector of independent parameters, dimensionality  $5 \times 1$ .

**Table 1**

Characteristics of horizontal and vertical position of wells

Parameters	No. of a well, date of carottage						
	337 09/ 2006	344 01/ 2008	546 03/ 2003	512 09/ 2004	506 06/ 2007	501 05/ 2004	352 06/ 2007
The height of the mouth, m	137.4	141.0	144.6	137	142	138.2	137.9
The height of the face, m	-3532	-3679	-3495	-3483	-3477	-3411	-3772
$x_{mouth}$ , m	6434	6104	8939	6391	5641	5984	6454
$y_{mouth}$ , m	8735	9491	9292	8478	10533	9457	8796

prediction.

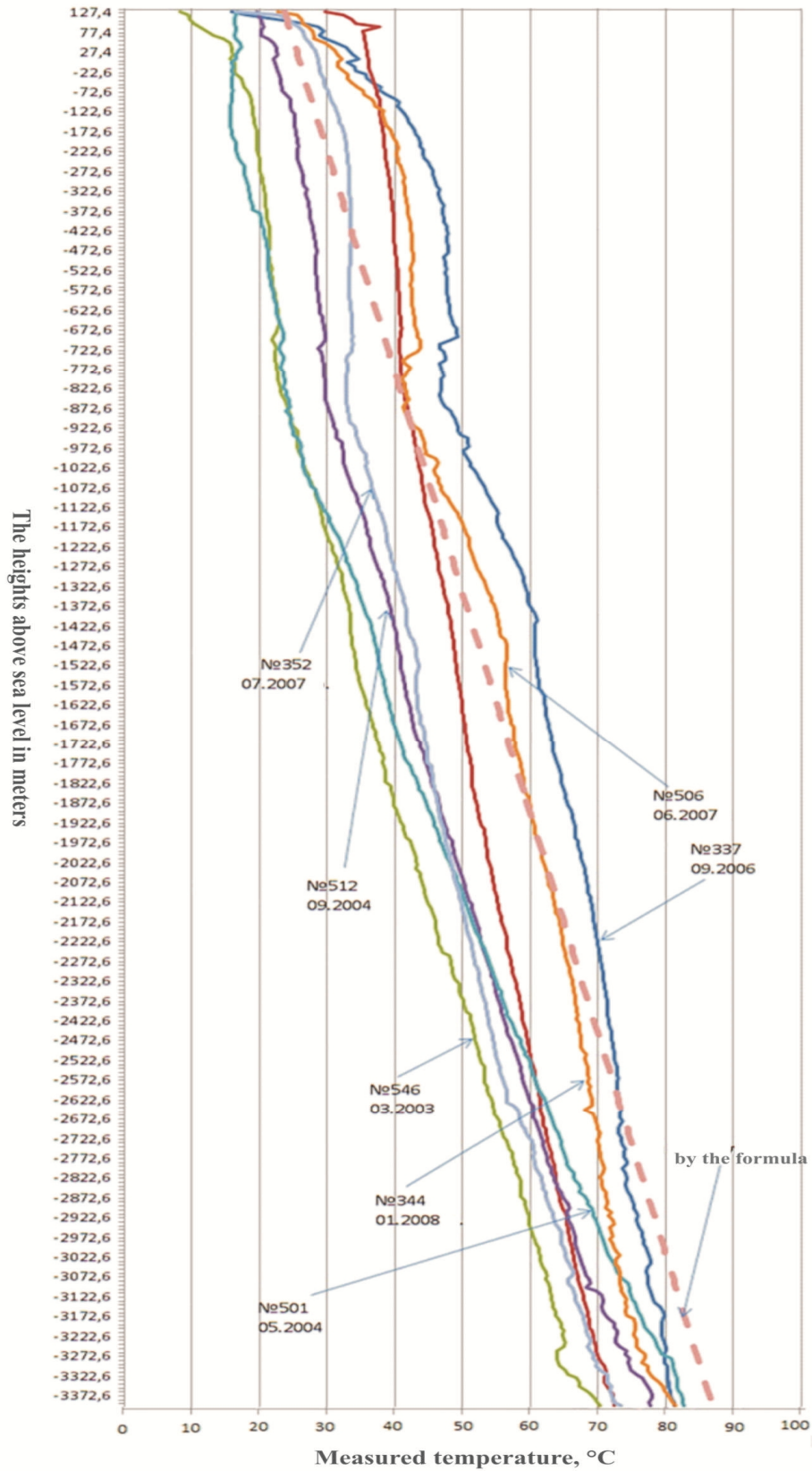


Fig. 1. Results of temperature measurements in the field.

To estimate the unknown vector of parameters, dependence (4) should not be considered as functional,

but as stochastic, which enables use the method of least squares for determination of parameters [4].

$$Q = Q(a) = \sum_{i=1}^k (y_i - \alpha_1 \cdot x_{i1} - \alpha_2 \cdot x_{i2} - \alpha_3 \cdot x_{i3} - \alpha_4 x_{i4} + b)^2 = \min \quad (5)$$

This approach allows not only find parameters of dependence (5), but also additional regression statistics for estimation of the validity of results, namely:

- a) coefficient of determination  $-r^2$ , which allows using F-statistics to estimate if established relationship is just random in the case of  $r^2$  is close to 1;
- b) values of estimates of standard errors of the

coefficients  $\alpha - S_{\alpha}$ , which allows to test the impact of each on the temperature.

For five wells of the second group according to measurements of temperature (2210 values of measured temperatures from the mouth to the face with the interval of 10 m) by dependences (2-5), the regression equation has been obtained:

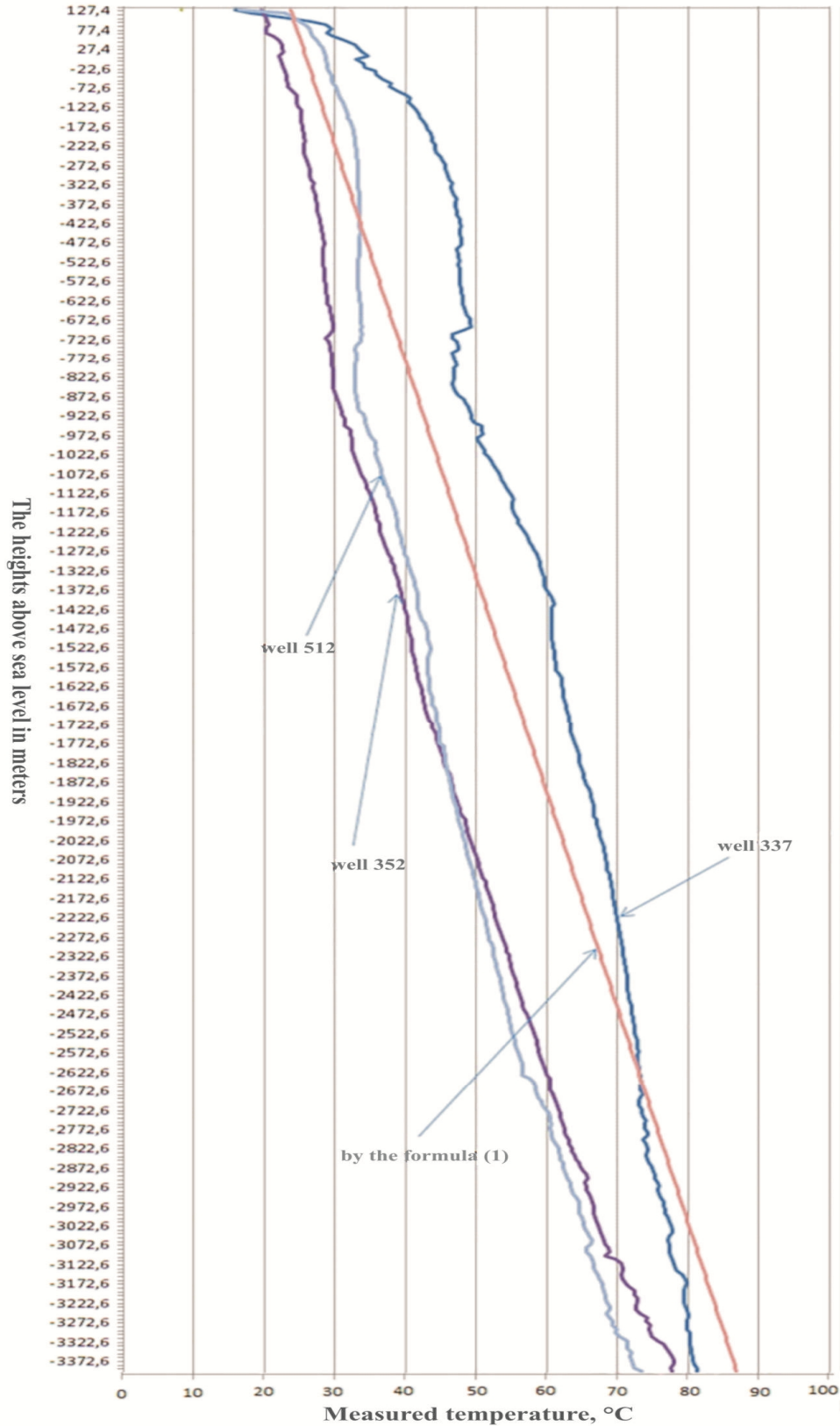


Fig. 2. Graph of temperature changes in three closely placed wells.

$$t_{i,static}(\text{indegrees}) = 16.4 + 0.122T - 0.0154H + 0.0005y - 0.00095x, \quad (6)$$

The values of spatial coordinates in the formula (6) are taken in meters,  $T$  – in months (in this case, from the date 03.2002), result is obtained in degrees. Note, that the equation is valid only for chosen conditional coordinate system.

Coefficient of determination is equal to 0.932, which indicates strong relationship between parameters and temperature in the wellbore.

The values of the standard errors of coefficients are equal to:  $S_{a1} = 0.005$ ,  $S_{a2} = 1E-4$ ,  $S_{a3} = 0.003$ ,  $S_{a4} = 0.0001$  and  $S_b = 2.7$ , which indicates possible temperature dependence on each of these factors, especially on  $H$ .

With the availability of relevant data, on the same technique, it is possible to predict temperature regime in the annulus –  $t_D$ .

Found value of  $S_{a,allow}$  to establish the effect of each of the coefficients  $a$  on the result. This requires finding the value for the given coefficient  $t = \frac{\alpha}{S_{a\alpha}}$  and comparing it with the critical value, which can be found using Microsoft Excel function TINV (СТЮДРАСПОБР). The corresponding values of the coefficients are equal  $t_T = 26$ ,  $t_H = 168$ ,  $t_y = 1.7$ ,  $t_x = 8.8$ ,  $t_b = 6$  at the critical value  $T.INV.2T(0.05, 2210) = 1.96$ . This allows to make confident conclusion that all parameters are useful for measuring temperature. For the given field, the depth and time of exploitation of a field make main influence on temperature. As for the parameters of the horizontal position, relatively small values of  $t$  indicate that the investigated wells are located on an area, where there is no significant lithologic and stratigraphic differentiation of rocks.

To compare the accuracy of approximation, sum of squared errors SSE of the measured values of temperature from obtained values for the same depths using equations (6) and (2) and the corresponding values of the mean squared errors have been calculated using the formula:

$$m = \sqrt{\frac{SSE}{n-1}},$$

where  $n$  is the total number of temperature measurements in all wells at all altitudes – 2210.

Obtained values of mean squared errors –  $m$  for multiple linear regression equation based on the

calculation results are equal to  $4.3^{\circ}\text{C}$ , and for equation (2) –  $12.7^{\circ}\text{C}$ , which indicates a significant increase of accuracy.

## Summary

For a field, which is in operation for a long time, measured temperature values in wells at the same altitudes may vary to  $20^{\circ}\text{C}$  in the absence of lithologic and stratigraphic differentiation of rocks.

The temperature in the wellbore during drilling depends on technogenic factors and differs noticeably from the temperature of the annulus, which depends mostly on exogenic factors.

Such temperature difference significantly affects on the stress-strain state of the rocks and, accordingly, the modes of drilling, and it should be considered when choosing the drilling and plugging solutions.

The temperature in the wellbore significantly correlates with not only the depth and lithologic and stratigraphic differentiation of rocks, but also with general increase of temperature of a field during its operation. Therefore, in the study of thermal processes during the drilling a field, using multiple regression equation, the parameters of which are defined using data of temperature measurements in a number of wells already drilled, more accurate data on the thermal regime can be obtained for the given field than using traditional methods.

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## Використання множинної лінійної регресії для прогнозу теплового режиму свердловин

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Розглядаються питання прогнозування теплового режиму свердловини для підрахунку параметрів буріння. Проведено аналіз досліджень і публікацій і відзначено, що не дивлячись на те, що на необхідність такого прогнозу вказується, рекомендацій, щодо методики виконання, в літературі немає. Пропонується виконувати такий прогноз з використанням багатофакторного регресійного аналізу, використовуючи результати вимірів температури, які виконують під час геофізичних досліджень свердловини відразу після закінчення буріння. Приведено приклад використання методики, що пропонується. На основі математико-статистичного аналізу одержаних результатів підтверджено, що температурний режим залежить не тільки від глибини, планового положення, але і від часу експлуатації родовища.

**Ключові слова:** родовище, свердловина, геотермічна ступінь, множинна лінійна регресія, прогноз.