

ON AUTOMORPHISMS OF SUPEREXTENSIONS OF SEMIGROUPS

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A family \mathcal{M} of non-empty subsets of a set X is called an *upfamily* if for each set $A \in \mathcal{M}$ any subset $B \supset A$ of X belongs to \mathcal{M} . By $v(X)$ we denote the set of all upfamilies on a set X . Each family \mathcal{B} of non-empty subsets of X generates the upfamily $\{A \subset X : \exists B \in \mathcal{B} (B \subset A)\}$ which we denote by $\langle B \subset X : B \in \mathcal{B} \rangle$. An upfamily \mathcal{F} that is closed under taking finite intersections is called a *filter*. A filter \mathcal{U} is called an *ultrafilter* if $\mathcal{U} = \mathcal{F}$ for any filter \mathcal{F} containing \mathcal{U} . The family $\beta(X)$ of all ultrafilters on a set X is called the *Stone-Čech compactification* of X , see [6]. An ultrafilter $\langle \{x\} \rangle$, generated by a singleton $\{x\}$, $x \in X$, is called *principal*. Each point $x \in X$ is identified with the principal ultrafilter $\langle \{x\} \rangle$ generated by the singleton $\{x\}$, and hence we can consider $X \subset \beta(X) \subset v(X)$. It was shown in [3] that any associative binary operation $* : S \times S \rightarrow S$ can be extended to an associative binary operation $* : v(S) \times v(S) \rightarrow v(S)$ by the formula

$$\mathcal{L} * \mathcal{M} = \left\langle \bigcup_{a \in L} a * M_a : L \in \mathcal{L}, \{M_a\}_{a \in L} \subset \mathcal{M} \right\rangle$$

for upfamilies $\mathcal{L}, \mathcal{M} \in v(S)$. In this case the Stone-Čech compactification $\beta(S)$ is a subsemigroup of the semigroup $v(S)$. The semigroup $v(S)$ contains as subsemigroups many other important extensions of S . In particular, it contains the semigroup $\lambda(S)$ of maximal linked upfamilies. An upfamily \mathcal{L} of subsets of S is said to be *linked* if $A \cap B \neq \emptyset$ for all $A, B \in \mathcal{L}$. A linked upfamily \mathcal{M} of subsets of S is *maximal linked* if \mathcal{M} coincides with each linked upfamily \mathcal{L} on S that contains \mathcal{M} . It follows that $\beta(S)$ is a subsemigroup of $\lambda(S)$. The space $\lambda(S)$ is well-known in General and Categorical Topology as the *superextension* of S , see [7].

Given a semigroup S we shall discuss the algebraic structure of the automorphism group $\text{Aut}(\lambda(S))$ of the superextension $\lambda(S)$ of S . We show that any automorphism of a semigroup S can be extended to an automorphism of its superextension $\lambda(S)$, and the automorphism group $\text{Aut}(\lambda(S))$ of the superextension $\lambda(S)$ of a semigroup S contains a subgroup, isomorphic to the group $\text{Aut}(S)$. We describe in [1, 2, 4] automorphism groups of superextensions of groups, finite monogenic semigroups, null semigroups, almost null semigroups, right zero semigroups, left zero semigroups and all three-element semigroups.

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