

triangular numbers. However, no other balancing-like sequence enjoys this property. Furthermore, the N -th triangular-like number of the sequence $BL(A, -1)$ is equal to the sum of first n terms of the sequence $BL(A^2 - 2, -1)$.

NEW FORMULAS FOR VIETA–JACOBSTHAL AND VIETA–JACOBSTHAL–LUCAS POLYNOMIALS

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In [3], authors defined Vieta–Jacobsthal and Vieta–Jacobsthal–Lucas polynomials by the recurrences $G_n(x) = G_{n-1}(x) - 2xG_{n-2}(x)$ and $g_n(x) = g_{n-1}(x) - 2xg_{n-2}(x)$, respectively, where $G_0(x) = 0$, $G_1(x) = 1$, $g_0(x) = 2$, $g_1(x) = 1$. Note that $G_n(-\frac{1}{2}) = F_n$ and $g_n(-\frac{1}{2}) = L_n$, where F_n and L_n are the n th Fibonacci and Lucas number.

We obtained some identities involving polynomials $G_n(x)$ and $g_n(x)$ and multinomial coefficients. Our approach is similar in spirit to [1, 2].

Let $|t| = t_1 + \dots + t_n$, $\tau_n = t_1 + 2t_2 + \dots + nt_n$, and $p_n(t) = \frac{(-1)^{|t|}|t|!}{t_1! \dots t_n!}$.

Theorem. *Let $n \geq 2$, except then noted otherwise. Then*

$$\begin{aligned} \sum_{\tau_n=n} p_n(t) G_2^{t_1}(x) G_3^{t_2}(x) \cdots G_{n+1}^{t_n}(x) &= 0, \quad n \geq 3, \\ \sum_{\tau_n=n} p_n(t) G_3^{t_1}(x) G_4^{t_2}(x) \cdots G_{n+2}^{t_n}(x) &= (2x)^n, \\ \sum_{\tau_n=n} p_n(t) G_3^{t_1}(x) G_5^{t_2}(x) \cdots G_{2n+1}^{t_n}(x) &= (-2x)^{n-1}, \end{aligned}$$

$$\sum_{\tau_n=n} p_n(t)g_2^{t_1}(x)g_3^{t_2}(x)\cdots g_{n+1}^{t_n}(x) = 2^{2n-3}(8x+1)x^{n-1},$$

$$\sum_{\tau_n=n} p_n(t)g_3^{t_1}(x)g_4^{t_2}(x)\cdots g_{n+2}^{t_n}(x) = 2(2x)^{n-1}((2^{n+1}-1)x+2^{n-2}),$$

$$\sum_{\tau_n=n} p_n(t)g_3^{t_1}(x)g_5^{t_2}(x)\cdots g_{2n+1}^{t_n}(x) = (8x+1)(2x)^{n-1},$$

where the summation is over integers $t_i \geq 0$ satisfying $\tau_n = n$.

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POSITIVITY OF CHARACTER SUM

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Let α be a positive real number. For any prime p let $L(\alpha, p)$ be the sum of first $\alpha * p$ Legendre symbols modulo p . It turns out that this quantity is in some sense positively biased. We will discuss several results concerning positivity of $L(\alpha, p)$ and connections between $L(\alpha, p)$ and random multiplicative functions.

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