formula in the space $\tilde{W}_2^{(m)}(T_n)$ is the only formula with the coefficients $\overset{o}{c}$ when both the nodes of the cubature formula are the lattice image on the torus T_n and whose coefficients are equal to each other $c_1 = c_2 = .. = c_N = \overset{o}{c}$, where

$$\overset{o}{c} = \frac{\widehat{P}_o + \frac{1}{(2\pi)^{2m}} \cdot \frac{1}{N^{2m}} \sum\limits_{k \neq 0} \frac{\widehat{P}_k}{|k|^{2m}}}{N\left(1 + \frac{1}{(2\pi)^{2m}} \frac{1}{N^{2m}} \sum\limits_{k \neq 0} \frac{1}{|k|^{2m}}\right)}. \tag{4}$$

In this case,

$$\left\| \overset{0}{\ell}_N(x) \Big/ \tilde{W}_2^{(m)^*}(T_n) \right\|^2 = \frac{A}{N^{2m}} + \frac{B}{N^{4m}}, \tag{5}$$

where

$$\left\| \overset{0}{\ell}_N(x) \Big/ \tilde{W}_2^{(m)^*}(T_n) \right\| = \inf_{c_\lambda, x^{(\lambda)}} \left\| \ell(x) \Big/ \tilde{W}_2^{(m)^*}(T_n) \right\|^2,$$

$$A = \frac{1}{D(2\pi)^{2m}} \sum_{k \neq 0} \frac{\left(\widehat{P}_k - P_o\right)^2}{|k|^{2m}},$$

$$B = \frac{1}{D(2\pi)^{4m}} \left[\sum_{k' \neq 0} \frac{1}{|k'|^{2m}} \sum_{k \neq 0} \frac{\widehat{P}_k^2}{|k|^{2m}} - \left(\sum_{k \neq 0} \frac{\widehat{P}_k}{|k|^{2m}} \right)^2 \right],$$

$$D = 1 + \frac{1}{(2\pi)^{2m}} \cdot \frac{1}{N^{2m}} \sum_{k \neq 0} \frac{1}{|k|^{2m}}.$$

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ON IDENTITIES FOR VIETA-FIBONACCI POLYNOMIALS USING TOEPLITZ-HESSENBERG MATRICES

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In [1], Horadam consider the Vieta-Fibonacci polynomials which are defined by the following recurrence relation

$$V_n(x) = xV_{n-1}(x) - V_{n-2}(x),$$

with $V_0(x) = 0$, $V_1(x) = 1$, for $n \geq 2$. In this paper, we investigate some families of Toeplitz-Hessenberg determinants (see, for example, [2, 3] and the bibliography given there) the entries of which are Vieta-Fibonacci polynomials with successive, even, and odd subscripts. This leads us to discover some new identities with multinomial coefficients for these polynomials. For example,

$$\sum_{\tau_n = n} p_n(t) V_0^{t_1}(x) V_1^{t_2}(x) \cdots V_{n-1}^{t_n}(x) = x^{n-2}, \qquad n \geq 2;$$

$$\sum_{\tau_n = n} (-1)^{T_n} p_n(t) V_1^{t_1}(x) V_3^{t_2}(x) \cdots V_{2n-1}^{t_n}(x) = x^2(1 - x^2)^{n-2}, \qquad n \geq 2;$$

$$\sum_{\tau_n = n} (-1)^{T_n} p_n(t) V_2^{t_1}(x) V_3^{t_2}(x) \cdots V_{n+1}^{t_n}(x) = 0, \qquad n \geq 3;$$

$$\sum_{\tau_n = n} (-1)^{T_n} p_n(t) V_3^{t_1}(x) V_5^{t_2}(x) \cdots V_{2n+1}^{t_n}(x) = (-1)^n x^2, \qquad n \geq 2;$$

$$\sum_{\tau_n = n} p_n(t) \left(\frac{V_0(x)}{x}\right)^{t_1} \left(\frac{V_2(x)}{x}\right)^{t_2} \cdots \left(\frac{V_{2n-2}(x)}{x}\right)^{t_n} = (x^2 - 2)^{n-2}, \qquad n \geq 2;$$

$$\sum_{\tau_n = n} (-1)^{T_n} p_n(t) \left(\frac{V_3(x)}{x}\right)^{t_1} \left(\frac{V_4(x)}{x}\right)^{t_2} \cdots \left(\frac{V_{n+2}(x)}{x}\right)^{t_n} = x^{-n}, \qquad n \geq 2;$$

$$\sum_{\tau_n = n} (-1)^{T_n} p_n(t) \left(\frac{V_4(x)}{x}\right)^{t_1} \left(\frac{V_6(x)}{x}\right)^{t_2} \cdots \left(\frac{V_{2n+2}(x)}{x}\right)^{t_n} = 0, \qquad n \geq 3,$$

where $\tau_n = t_1 + 2t_2 + \cdots + nt_n$, $T_n = t_1 + \cdots + t_n$, $p_n(t) = \frac{(t_1 + \cdots + t_n)!}{t_1! \cdots t_n!}$ is the multinomial coefficient, and the summation is over nonnegative integers satisfying $\tau_n = n$.

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OPTIMAL QUADRATURE FORMULAS WITH DERIVATIVE IN $W_2^{(2,1)}(0,1)$ SPACE

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We consider the following quadrature formula

$$\int_0^1 \varphi(x)dx \cong \sum_{\beta=0}^{N} \left(C_0[\beta]\varphi(h\beta) + C_1[\beta]\varphi'(h\beta)\right) \quad (1)$$

where $[\beta] = (h\beta)$, $h = 1/N$, N is a natural number $C_0[0] = C_0[N] = h/2$, and $C_0[\beta] = h$ for $\beta = 1, 2, 3, ..., N-1$, $C_1[\beta]$ are unknown coefficients of the formula (1), φ an element of the Hilbert space $W_2^{(2,1)}(0,1)$ equipped with the norm $\|\varphi\| = \sqrt{\int_0^1 (\varphi''(x) + \varphi'(x))^2 dx}$.

The error

$$(\ell, \varphi) = \int_0^1 \varphi(x)dx - \sum_{\beta=0}^{N} \left(C_0[\beta]\varphi(h\beta) + C_1[\beta]\varphi'(h\beta)\right)$$

of the formula (1) is estimated by the norm of the error functional ℓ in the conjugate space $W_2^{(2,1)*}(0,1)$, i.e.

$$\left\|\ell | W_2^{(2,1)*}(0,1)\right\| = \sup_{\left\|\varphi | W_2^{(2,1)}(0,1)\right\|=1} |(\ell, \varphi)|.$$

Furthermore, the norm of the error functional ℓ depends on the coefficients $C_1[\beta]$. We minimize the norm of the functional ℓ by coefficients, i.e. we find the following quantity

$$\left\|\mathring{\ell} | W_2^{(2,1)*}(0,1)\right\| = \inf_{C_1[\beta]} \left\|\ell | W_2^{(2,1)*}(0,1)\right\| \quad (3)$$

If $\left\|\mathring{\ell} | W_2^{(2,1)*}(0,1)\right\|$ is found then the functional is said to be correspond to the optimal quadrature formula (1) in $W_2^{(2,1)}(0,1)$ and the corresponding coefficients are called optimal.

Thus in order to construct optimal quadrature formulas of the form (1) we get the following problems.

Problem 1. Find the norm of the error functional ℓ in the space $W_2^{(2,1)*}(0,1)$.

Problem 2. Find the coefficients $C_1[\beta]$ which satisfy the equality (3).

Here we solve Problems 1 and 2 and the main result of the paper is the following.

Theorem. *The coefficients of optimal quadrature formulas of the form (1) in the space $W_2^{(2,1)}(0,1)$ have the following form*

$$\begin{cases} C_1[0] = \frac{h(e^h+1)}{2(e^h-1)} - 1, \\ C_1[\beta] = 0, \quad \beta = \overline{1, N-1} \\ C_1[N] = 1 - \frac{h(e^h+1)}{2(e^h-1)}. \end{cases},$$

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AN ALGORITHM FOR CONSTRUCTING LATTICE OPTIMAL INTERPOLATION FORMULAS IN A PERIODIC SPACE S.L. SOBOLEV $\tilde{W}_2^{(m)}(T_1)$.

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The problem of constructing interpolation formulas is one of the classical problems of computational mathematics and numerical analysis. Consider an interpolation formula of the form

$$f(x) \cong P_f(x) = \sum_{\beta=1}^{N} C_\beta(x) f(x^{(\beta)}), \quad (1)$$

with the error functional

$$\ell(x) = \delta(x-z) - \sum_{\beta=0}^{N} C_\beta(z)\delta(x - x^{(\beta)}). \quad (2)$$

over the space of S.L. Sobolev $\tilde{W}_2^{(m)}[0,1]$. Here, respectively, $c_\beta(z)$ and $x^{(\beta)}$ are the coefficients and nodes of the interpolation formula (1), $f(x) \in \tilde{W}_2^{(m)}[0,1]$. **Definition 1.** The space $\tilde{W}_2^{(m)}(T_1)$ is defined as the space of functions of given one-dimensional T_1-circle of length equal to one and having all generalized derivatives of order m that are square-summable in the norm [1].

$$\left\| f \big/ \tilde{W}_2^{(m)}(T_1) \right\|^2 = \left(\int_{T_1} f(x)\, dx \right)^2 + \sum_{k \neq 0} |2\pi k|^{2m} \left| \hat{f}_k \right|^2. \quad (3)$$