Definition 3. Suppose that M is an L-structure and $\varphi(\bar{v})$ is an L_M-formula. We will define $RM_M(\varphi)$ the Morley rank of φ in M. First, we inductively define $RM_M(\varphi) \geq \alpha$ for α an ordinal:

i) $RM_M(\varphi) \geq 0$ if and only if $\varphi(M)$ is non empty;

ii) if α is limit ordinal, then $RM_M(\varphi) \geq \alpha$ if and only if $RM_M(\varphi) \geq \beta$, for all $\beta < \alpha$;

iii) for any ordinal α, $RM_M(\varphi) \geq \alpha+1$ if and only if there are L_M-formulas $\psi_1(\bar{v}), \psi_2(\bar{v}), \ldots$, such that $\psi_1(M), \psi_2(M), \ldots$ is an infinite family of pairwise disjoint subsets of $\varphi(M)$ and $RM_M(\psi_i) \geq \alpha$ for all i.

Theorem. On the type of the finite Morley rank holds exchange principle.

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ON JACOBSTHAL AND JACOBSTHAL–LUCAS IDENTITIES WITH MULTINOMIAL COEFFICIENTS

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The *Jacobsthal* and *Jacobsthal–Lucas* sequences $\{J_n\}_{n\geq 0}$ and $\{j_n\}_{n\geq 0}$ are defined by the recurrence relations $J_{n+1} = J_n + 2J_{n-1}$, $J_0 = 0$, $J_1 = 1$, and $j_{n+1} = j_n + 2j_{n-1}$, $j_0 = 2$, $j_1 = 1$, for $n \geq 1$. The numbers J_n appear as the integer sequence A001045 from [6] while the numbers j_n is A014551. Jacobsthal and Jacobsthal–Lucas sequences have a rich history and many remarkable properties (see [1–5] for more details).

Theorem 1. *Let $n \geq 1$, except when noted otherwise. The following formulas hold:*

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) J_0^{t_1} J_1^{t_2} \cdots J_{n-1}^{t_n} = -F_{n-1},$$

$$\sum_{(t_1,\ldots,t_n)} p_n(t) J_0^{t_1} J_1^{t_2} \cdots J_{n-1}^{t_n} = \frac{\sqrt{13}}{13} \left(\left(\frac{1+\sqrt{13}}{2} \right)^{n-1} - \left(\frac{1-\sqrt{13}}{2} \right)^{n-1} \right),$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) J_0^{t_1} J_2^{t_2} \cdots J_{2n-2}^{t_n} = \frac{\sqrt{5}}{5} \left(\left(\frac{5-\sqrt{5}}{2} \right)^{n-1} - \left(\frac{5+\sqrt{5}}{2} \right)^{n-1} \right),$$

$$\sum_{(t_1,\ldots,t_n)} p_n(t) J_0^{t_1} J_2^{t_2} \cdots J_{2n-2}^{t_n} = \frac{\sqrt{13}}{13} \left(\left(\frac{5+\sqrt{13}}{2} \right)^{n-1} - \left(\frac{5-\sqrt{13}}{2} \right)^{n-1} \right),$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) J_1^{t_1} J_2^{t_2} \cdots J_n^{t_n} = \left((-1)^n - 1 \right) \cdot 2^{\frac{n-3}{2}},$$

$$\sum_{(t_1,\ldots,t_n)} p_n(t) J_1^{t_1} J_2^{t_2} \cdots J_n^{t_n} = \frac{\sqrt{3}}{6} \left((1+\sqrt{3})^n - (1-\sqrt{3})^n \right),$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) J_1^{t_1} J_3^{t_2} \cdots J_{2n-1}^{t_n} = -\frac{1}{2} \left((2+\sqrt{2})^{n-1} + (2-\sqrt{2})^{n-1} \right),$$

$$\sum_{(t_1,\ldots,t_n)} p_n(t) J_1^{t_1} J_3^{t_2} \cdots J_{2n-1}^{t_n} = \frac{1}{6} \left((3+\sqrt{3})^n + (3-\sqrt{3})^n \right),$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) J_2^{t_1} J_3^{t_2} \cdots J_{n+1}^{t_n} = 0, \qquad n \geq 3,$$

$$\sum_{(t_1,\ldots,t_n)} p_n(t) J_2^{t_1} J_3^{t_2} \cdots J_{n+1}^{t_n} = 2^{n-1} F_{n+1}, \qquad n \geq 3,$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) J_2^{t_1} J_4^{t_2} \cdots J_{2n}^{t_n} = -2^{n-1} n,$$

$$\sum_{(t_1,\ldots,t_n)} p_n(t) J_2^{t_1} J_4^{t_2} \cdots J_{2n}^{t_n} = \frac{\sqrt{5}}{10} \left((3+\sqrt{5})^n - (3-\sqrt{5})^n \right),$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) J_3^{t_1} J_4^{t_2} \cdots J_{n+2}^{t_n} = (-2)^n, \qquad n \geq 2,$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) J_3^{t_1} J_5^{t_2} \cdots J_{2n+1}^{t_n} = -2^{n-1}, \qquad n \geq 2,$$

$$\sum_{(t_1,\ldots,t_n)} p_n(t) J_3^{t_1} J_5^{t_2} \cdots J_{2n+1}^{t_n} = \frac{1}{16} \left((4+\sqrt{8})^{n+1} + (4-\sqrt{8})^{n+1} \right),$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) J_4^{t_1} J_5^{t_2} \cdots J_{n+3}^{t_n} = -(-2)^{n-1}(2n+3),$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) J_4^{t_1} J_6^{t_2} \cdots J_{2n+2}^{t_n} = 0, \qquad n \geq 3,$$

where the summation is over integers $t_i \geq 0$ *satisfying* $t_1 + 2t_2 + \cdots + nt_n = n$, $T = t_1 + \cdots + t_n$, $p_n(t) = \frac{(t_1+\cdots+t_n)!}{t_1!\cdots t_n!}$ *is multinomial coefficient, and* F_n *is the* n*-th Fibonacci number (sequence A000045 in [6]).*

Theorem 2. *Let* $n \geq 1$*, except when noted otherwise. The following formulas hold:*

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) j_0^{t_1} j_1^{t_2} \cdots j_{n-1}^{t_n} = \frac{\sqrt{13}-11}{6\sqrt{13}} \left(\frac{\sqrt{13}-1}{2}\right)^n + \frac{\sqrt{13}+11}{6\sqrt{13}} \left(\frac{-\sqrt{13}-1}{2}\right)^n,$$

$$\sum_{(t_1,\ldots,t_n)} p_n(t) j_0^{t_1} j_1^{t_2} \cdots j_{n-1}^{t_n} = \frac{7\sqrt{13}-13}{26} \left(\frac{3+\sqrt{13}}{2}\right)^n - \frac{7\sqrt{13}+13}{26} \left(\frac{3-\sqrt{13}}{2}\right)^n,$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) j_0^{t_1} j_2^{t_2} \cdots j_{n-2}^{t_n} = \frac{19\sqrt{13}+65}{26} \left(\frac{\sqrt{13}-3}{2}\right)^n - \frac{19\sqrt{13}-65}{26} \left(\frac{-\sqrt{13}-3}{2}\right)^n,$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) j_1^{t_1} j_2^{t_2} \cdots j_n^{t_n} = (-1)^{\lfloor \frac{3n}{2} \rfloor} \cdot \frac{(4+\sqrt{2})(\sqrt{2})^n + (4-\sqrt{2})(-\sqrt{2})^n}{4},$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) j_1^{t_1} j_3^{t_2} \cdots j_{2n-1}^{t_n} = 9(-1)^{\lfloor \frac{4n-1}{2} \rfloor} \cdot \Big((2-i\sqrt{2})^{n-3} + (2+i\sqrt{2})^{n-3}\Big),$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) j_2^{t_1} j_3^{t_2} \cdots j_{n+1}^{t_n} = \frac{9(-4)^n}{8}, \qquad n \geq 2,$$

$$\sum_{(t_1,\ldots,t_n)} p_n(t) j_2^{t_1} j_4^{t_2} \cdots j_{2n}^{t_n} = 2^{n-2}\big((-1)^n 9 - 1\big),$$

$$\sum_{(t_1,\ldots,t_n)} (-1)^T p_n(t) j_3^{t_1} j_5^{t_2} \cdots j_{2n+1}^{t_n} = -9(-2)^{n-1}, \qquad n \geq 2,$$

where the summation is over integers $t_i \geq 0$ *satisfying* $t_1 + 2t_2 + \cdots + nt_n = n$, $T = t_1 + \cdots + t_n$, $p_n(t) = \frac{(t_1+\cdots+t_n)!}{t_1!\cdots t_n!}$, $i = \sqrt{-1}$, *and* $\lfloor \alpha \rfloor$ *is the floor of* α.

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UNIVERSAL ENVELOPING BICOMMUTATIVE ALGEBRAS FOR METABELIAN LIE ALGEBRAS

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We establish Poincare-Birkhoff-Witt theorem (PBW theorem) for metabelian Lie algebras and Bicommutative algebras.

Let $A = (A, \circ)$ be an algebra over field with characteristic $p \geq 0$ and $A \times A \to A$, $(a, b) \mapsto a \circ b$, is product. An algebra $(A, \circ)$ is called *Bicommutative*, if

$$(a \circ b) \circ c = (a \circ c) \circ b (RCom)$$

$$a \circ (b \circ c) = b \circ (a \circ c) (LCom)$$

for any $a, b, c \in A$. We mention that bicommutative algebra over new commutator operation $[a, b] = a \circ b - b \circ a$ satisfies the following three equations:

$$1. [a, b] = -[b, a]$$

$$2. [[a, b], c] + [[b, c], a] + [[c, a], b] = 0$$

$$3. [[a, b], [c, d]] = 0.$$

It means bicommutative algebra over new commutator operation is Lie metabelian algebra. Let us remain that an algebra A is called We use description of linear basis of algebras metabelian Lie algebras. Let $F = F < a_1, \cdots, a_r >$ be free Lie metabelian algebra generating on free variables $a_1, \cdots, a_r$. Then linear basis of F will be the following right-normed products $[[[a_{i_1}, a_{i_2}], a_{i_3}], \cdots, a_{i_k}]$ where $i_1 > i_2 \leq i_3 \leq i_k$

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