GREEN'S FUNCTION FOR DEGENERATE HIGH-ORDER PARABOLIC OPERATORS

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Let's denote $n_j \in N,\ j = \overline{1,p},\ n_1 \geq n_2 \geq \dots \geq n_p,\ n_0 = \sum_{j=1}^{p} n_j,\ x = (x_1, \dots, x_p),\ x_j = (x_{j1}, \dots, x_{jn_j}),\ x_j \in R^{n_j},\ x \in R^{n_0},\ \xi = (\xi_1, \dots, \xi_p),\ \xi_j = (\xi_{j1}, \dots, \xi_{jn_j}), \xi_j \in R^{n_j}, \xi \in R^{n_0}, x^{(j)} = (x_1, \dots, x_j) \in R^{\sum_{k=1}^{j} n_k}, \xi^{(j)} = (\xi_1, \dots, \xi_j) \in R^{\sum_{k=1}^{j} n_k},\ j = \overline{2,p}.$

$$x_j - \xi_j + \sum_{k=1}^{j-1} x_k \frac{(t-\tau)^{j-k}}{(j-k)!} = \left(x_{j1} - \xi_{j1} + \sum_{k=1}^{j-1} x_{k1} \frac{(t-\tau)^{j-k}}{(j-k)!}, .., \right.$$

$$\left. x_{jn_j} - \xi_{jn_j} + \sum_{k=1}^{j-1} x_{kn_j} \frac{(t-\tau)^{j-k}}{(j-k)!} \right),$$

$\Gamma(\alpha)$ – gamma Euler's function.

$$\rho_1(t, x_1, \tau, \xi_1) = \left(|x_1 - \xi_1| (t-\tau)^{-\frac{1}{2b}} \right)^q,\ q = \frac{2b}{2b-1},\ b \in N,$$

$$\rho_j\left(t, x^{(j)}, \tau, \xi^{(j)}\right) = \left(\left| x_j - \xi_j + \sum_{k=1}^{j-1} x_k \frac{(t-\tau)^{j-k}}{(j-k)!} \right| (t-\tau)^{-\left(j-1+\frac{1}{2b}\right)} \right)^q,\ j = \overline{2,p}.$$

$$\xi(t,\tau) = \left(\xi_1, \xi_2 - \xi_1\ (t-\tau), \dots,\ \xi_p + \sum_{k=1}^{p-1} (-1)^{p-k} \xi_k \frac{(t-\tau)^{p-k}}{(p-k)!} \right).$$

We investigate the Cauchy problem for the equation:

$$\partial_t u(t,x) - \sum_{j=1}^{p-1} \sum_{\mu=1}^{n_{j+1}} x_{j\mu} \partial_{x_{j+1}\mu} u(t,x) = \sum_{|k| \leq 2b} a_k(t,x) D_{x_1}^k u(t,x), \tag{1}$$

with the initial condition

$$u(t,x) \Big|_{t=\tau} = u_0(x), \quad 0 \leq \tau \leq t \leq T, \tag{2}$$

where τ – fixed number, and operator

$$\partial_t - \sum_{|k| \leq 2b} a_k(t,x) D_{x_1}^k, \qquad D_{x_1}^k = \frac{(-1)^k \partial^{k_1 + \dots + k_{n_1}}}{\partial x_1^{k_1} \dots \partial x_{n_1}^{k_{n_1}}},\ |k| = k_1 + \dots + k_{n_1}, \tag{3}$$

uniformly parabolic in the sense of Petrovsky in the strip $\Pi_{[0,T]} = (t,x), x \in R^{n_0}, 0 \leq t \leq T$.

Let's suppose that

1) $a_k(t,x),\ \partial_{x_j} a_k(t,x),\ j = \overline{2,p}$, continuous and limited in $\Pi_{[0,T]}$.
2) There are steals $\alpha \in (0,1],\ r \in (0,1]$ such that for any $x \in R^{n_0}, \xi \in R^{n_0}$ and $t \in [0,T]$,

$$|a_k(t,x) - a_k(t,\xi)| \leq c_1 \left(|x_1 - \xi_1|^\alpha + \sum_{j=2}^{p} |x_j - \xi_j| \right),$$

$$\left| \partial_{x_j} a_k(t,x) - \partial_{x_j} a_k(t,\xi) \right| \leq c_1 |x - \xi|^r,\ j = \overline{2,p}.$$

Theorem 1. *If conditions 1) -2) are satisfied, then equation* (1) *has a fundamental solution to the Cauchy problem* (1)-(2) $Z(t,x;\tau,\xi)$ *at* $t>\tau$ *and correct grades:*

$$\left|\partial_{x_j} Z(t,x;\tau,\xi)\right| \le A(t-\tau)^{-\sum_{s=1}^{p}\frac{2b(s-1)+1}{2b}(n_s+|m_s|)}\Phi\left(t,x;\tau,\xi\right),$$

$m_s=0,$ *at* $s\neq j,\ m_j=1,\ j=\overline{2,p}.$

$$\left|\partial_{x_1}^{m_1} Z(t,x;\tau,\xi)\right| \le A_{m_1}(t-\tau)^{-\frac{n_1+|m_1|}{2b}-\sum_{s=2}^{p}\frac{2b(s-1)+1}{2b}n_s}\Phi\left(t,x;\tau,\xi\right),$$

$|m_1|\le 2b,\ x\in R^{n_0},\quad \xi\in R^{n_0},\quad 0\le\tau<t\le T,$ *where*

$$\Phi\left(t,x;\tau,\xi\right)=\sum_{=1}^{\infty}A^s\Gamma\left(1+\frac{s\alpha^*}{2b}\right)\Gamma\left(\frac{\alpha^*}{2\mathrm{b}}\right)\Gamma^{-1}\left(1+\frac{\alpha^*\left(1+s\right)}{2b}\right)\exp\{-c_0$$

$$\rho_1\left(t,x_1,\tau,\xi_1\right)-2^{-2sp}c_0\sum_{j=2}^{p}\rho_j\left(t,x^{(j)},\tau,\xi^{(j)}\right)\},$$

positive constants A, A_{m_1}, c_0 *depend on* $n_0,\ 2b,\ c_1,\alpha,r$ *and the constant parabolicity of the operator* (3), $\sup_{(t,x)\in\Pi_{[0,T]}}|a_k(t,x)|$, $\alpha^*=\min\alpha,r$.

References

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